

Problem A.1

(a) $\int_0^3 (2x^2 + 7x + 3)\delta(x - 1) dx = ?$

(b) $\int_0^3 \ln(1 + x)\delta(\pi - x) dx = ?$

Solution**Part (a)**

Since $x = 1$ lies within the integration interval, $0 < x < 3$, the delta function sifts the value of $(2x^2 + 7x + 3)$ evaluated at $x = 1$.

$$\begin{aligned}\int_0^3 (2x^2 + 7x + 3)\delta(x - 1) dx &= (2x^2 + 7x + 3)\Big|_{x=1} \\ &= 2(1)^2 + 7(1) + 3 \\ &= 12\end{aligned}$$

Part (b)

Since $x = \pi$ lies outside the integration interval, $0 < x < 3$, the delta function is zero.

$$\begin{aligned}\int_0^3 \ln(1 + x)\delta(\pi - x) dx &= \int_0^3 \ln(1 + x) \left[\frac{1}{|-1|} \delta(x - \pi) \right] dx \\ &= \int_0^3 \ln(1 + x)\delta(x - \pi) dx \\ &= \int_0^3 \ln(1 + x)(0) dx \\ &= 0\end{aligned}$$